Three-point Correlation Function of Giant Magnons in the Lunin-Maldacena background

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Abstract

We compute semiclassical three-point correlation function, or structure constant, of two finite-size (dyonic) giant magnon string states and a light dilaton mode in the Lunin-Maldacena background, which is the γ-deformed, or TsT-transformed $AdS_5 \times S^5_\gamma$, dual to $\mathcal{N} = 1$ super Yang-Mills theory. We also prove that an important relation between the structure constant and the conformal dimension, checked for the $\mathcal{N} = 4$ super Yang-Mills case, still holds for the γ-deformed string background.

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1 Introduction

As is well known, the correlation functions of any conformal field theory can be determined in principle in terms of the basic conformal data $\{\Delta_i, C_{ijk}\}$, where Δ_i are the conformal dimensions defined by the two-point correlation functions

$$\left\langle \mathcal{O}_i^\dagger(x_1)\mathcal{O}_j(x_2)\right\rangle = \frac{C_{12}\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}} \tag{1.1}$$

and C_{ijk} are the structure constants in the OPE

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}. \tag{1.2}$$

Thus, the determination of the initial conformal data for a given CFT is the most important step in the conformal bootstrap approach.

In view of the AdS/CFT duality [1], between strings on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills (SYM) theory, the correlators of single-trace conformal primary operators on the gauge theory side, in the planar limit, should be related to the correlation functions of the corresponding closed-string vertex operators. The conformal dimension Δ can be expressed in terms of the conserved charges and the string tension by the marginality condition on the vertex operator.

In particular, the three-point functions of two heavy operators and a light dilaton operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration:

$$\langle V_H(x_1)V_H(x_2)V_L(x_3)\rangle = V_L(x_3)_{\text{classical}}.$$

For $|x_1| = |x_2| = 1$, $x_3 = 0$, the correlation function reduces to

$$\langle V_H(x_1)V_H(x_2)V_L(0)\rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the normalized structure constants

$$\mathcal{C}_3 = \frac{C_{123}}{C_{12}}$$

can be found from

$$\mathcal{C}_3 = c_\Delta V_L(0)_{\text{classical}}, \tag{1.3}$$

where c_Δ is the normalized constant of the corresponding light vertex operator.

Recently, there has been an impressive progress in the semiclassical calculations of two, three, and four-point functions with two heavy operators [2]-[22]. Almost all of these achievements are in the framework of the duality between string theory in $AdS_5 \times S^5$ and $\mathcal{N}=4$ SYM. An exception is the paper [22], considering the case of strings on Lunin-Maldacena background [23], dual to $\mathcal{N}=1$ SYM in four dimensions. In particular, the three-point correlation function of two infinite-size giant magnons and the dilaton has been obtained there. Our aim here is to generalize this result to the case of *finite-size* dyonic giant magnons.

2 Three-point correlation function

The bosonic part of the Green-Schwarz action for strings on the γ_i-deformed $AdS_5 \times S^5_{\gamma_i}$ [24] reduced to $R_t \times S^5_{\gamma_i}$ can be written as (the common radius R of AdS_5 and $S^5_{\gamma_i}$ is set to 1)

$$\begin{aligned} S &= -\frac{T}{2}\int d\tau d\sigma \left\{\sqrt{-\gamma}\gamma^{ab}\left[-\partial_a t\partial_b t + \partial_a r_i\partial_b r_i + G r_i^2\partial_a\phi_i\partial_b\phi_i \right.\right. \\ &\quad \left. + \, G r_1^2 r_2^2 r_3^2\left(\tilde{\gamma}_i\partial_a\phi_i\right)\left(\tilde{\gamma}_j\partial_b\phi_j\right)\right] \\ &\quad \left. -2G\,\epsilon^{ab}\left(\tilde{\gamma}_3 r_1^2 r_2^2\partial_a\phi_1\partial_b\phi_2 + \tilde{\gamma}_1 r_2^2 r_3^2\partial_a\phi_2\partial_b\phi_3 + \tilde{\gamma}_2 r_3^2 r_1^2\partial_a\phi_3\partial_b\phi_1\right)\right\}, \end{aligned} \tag{2.1}$$

where T is the string tension, γ^{ab} is the worldsheet metric, ϕ_i are the three isometry angles of the deformed $S^5_{\gamma_i}$, and

$$\sum_{i=1}^{3} r_i^2 = 1, \quad G^{-1} = 1 + \tilde{\gamma}_3^2 r_1^2 r_2^2 + \tilde{\gamma}_1^2 r_2^2 r_3^2 + \tilde{\gamma}_2^2 r_1^2 r_3^2. \tag{2.2}$$

The deformation parameters $\tilde{\gamma}_i$ are related to γ_i which appear in the dual gauge theory as follows

$$\tilde{\gamma}_i = 2\pi T\gamma_i = \sqrt{\lambda}\gamma_i.$$

When $\tilde{\gamma}_i = \tilde{\gamma}$ this becomes the supersymmetric background of [23], and the deformation parameter γ enters the $\mathcal{N}=1$ SYM superpotential in the following way

$$W \propto tr\left(e^{i\pi\gamma}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\gamma}\Phi_1\Phi_3\Phi_2\right).$$

This is the case we are going to consider here.

We restrict ourselves to the subspace $R_t \times S^3_\gamma$, parameterize (see (2.2))

$$r_1 = \sin\theta, \quad r_2 = \cos\theta,$$

and use the ansatz [25]

$$t(\tau,\sigma)=\kappa\tau, \quad \theta(\tau,\sigma)=\theta(\xi), \quad \phi_j(\tau,\sigma)=\omega_j\tau+f_j(\xi), \tag{2.3}$$
$$\xi=\alpha\sigma+\beta\tau, \quad \kappa,\omega_j,\alpha,\beta=\text{constants}, \quad j=1,2.$$

Then the string Lagrangian in conformal gauge, on the γ-deformed three-sphere, can be written as (prime is used for $d/d\xi$)

$$\mathcal{L}_\gamma=(\alpha^2-\beta^2)\left[\theta'^2+G\sin^2\theta\left(f_1'-\frac{\beta\omega_1}{\alpha^2-\beta^2}\right)^2+G\cos^2\theta\left(f_2'-\frac{\beta\omega_2}{\alpha^2-\beta^2}\right)^2\right.$$
$$\left.-\frac{\alpha^2}{(\alpha^2-\beta^2)^2}G\left(\omega_1^2\sin^2\theta+\omega_2^2\cos^2\theta\right)+2\alpha\tilde{\gamma}G\sin^2\theta\cos^2\theta\frac{\omega_2 f_1'-\omega_1 f_2'}{\alpha^2-\beta^2}\right], \tag{2.4}$$

where

$$G=\frac{1}{1+\tilde{\gamma}^2\sin^2\theta\cos^2\theta}.$$

The equations of motion for $f_{1,2}$ following from (2.4) can be integrated once to give

$$f_1'=\frac{1}{\alpha^2-\beta^2}\left[\frac{C_1}{\sin^2\theta}+\beta\omega_1-\tilde{\gamma}\left(\alpha\omega_2-\tilde{\gamma}C_1\right)\cos^2\theta\right], \tag{2.5}$$
$$f_2'=\frac{1}{\alpha^2-\beta^2}\left[\frac{C_2}{\cos^2\theta}+\beta\omega_2+\tilde{\gamma}\left(\alpha\omega_1+\tilde{\gamma}C_2\right)\sin^2\theta\right],$$

where $C_{1,2}$ are integration constants.

Replacing (2.5) into the Virasoro constraints one finds the first integral θ' of the equation of motion for θ and a relation among the parameters

$$\theta'^2=\frac{1}{(\alpha^2-\beta^2)^2}\left[(\alpha^2+\beta^2)\kappa^2-\frac{C_1^2}{\sin^2\theta}-\frac{C_2^2}{\cos^2\theta}\right. \tag{2.6}$$
$$\left.-\left(\alpha\omega_1+\tilde{\gamma}C_2\right)^2\sin^2\theta-\left(\alpha\omega_2-\tilde{\gamma}C_1\right)^2\cos^2\theta\right],$$
$$\omega_1C_1+\omega_2C_2+\beta\kappa^2=0. \tag{2.7}$$

Now, we introduce the variable

$$\chi=\cos^2\theta,$$

and the parameters

$$v = -\frac{\beta}{\alpha}, \quad u = \frac{\Omega_2}{\Omega_1}, \quad W = \left(\frac{\kappa}{\Omega_1}\right)^2, \quad K = \frac{C_2}{\alpha\Omega_1},$$
$$\Omega_1 = \omega_1\left(1 + \tilde{\gamma}\frac{C_2}{\alpha\omega_1}\right), \quad \Omega_2 = \omega_2\left(1 - \tilde{\gamma}\frac{C_1}{\alpha\omega_2}\right).$$

By using them and (2.7), the three first integrals can be rewritten as

$$\begin{aligned} f_1' &= \frac{\Omega_1}{\alpha}\frac{1}{1-v^2}\left[\frac{vW - uK}{1-\chi} - v(1-\tilde{\gamma}K) - \tilde{\gamma}u\chi\right], \\ f_2' &= \frac{\Omega_1}{\alpha}\frac{1}{1-v^2}\left[\frac{K}{\chi} - uv(1-\tilde{\gamma}K) - \tilde{\gamma}v^2W + \tilde{\gamma}(1-\chi)\right], \\ \theta' &= \frac{\Omega_1}{\alpha}\frac{\sqrt{1-u^2}}{1-v^2}\sqrt{\frac{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}{\chi(1-\chi)}}, \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} \chi_p + \chi_m + \chi_n &= \frac{2-(1+v^2)W-u^2}{1-u^2}, \\ \chi_p\chi_m + \chi_p\chi_n + \chi_m\chi_n &= \frac{1-(1+v^2)W+(vW-uK)^2-K^2}{1-u^2}, \\ \chi_p\chi_m\chi_n &= -\frac{K^2}{1-u^2}. \end{aligned} \tag{2.9}$$

We are interested in the case of finite-size giant magnons, which corresponds to

$$0 < \chi_m < \chi < \chi_p < 1, \quad \chi_n < 0.$$

Replacing (2.8) and (2.9) in (2.4), we find the final form of the Lagrangian to be (we set $\alpha = \Omega_1 = 1$ for simplicity)

$$\mathcal{L}_f = -\frac{1}{1-v^2}\left[2-(1+v^2)W - 2\tilde{\gamma}K - 2\left(1-\tilde{\gamma}K - u\left(u - \tilde{\gamma}uK + \tilde{\gamma}vW\right)\right)\chi\right].$$

To obtain the finite-size effect on the three-point correlator, we use (1.3) and the explicit expression for the dilaton vertex

$$V^d = (Y_4+Y_5)^{-4}\left[z^{-2}\left(\partial_+x_m\partial_-x^m + \partial_+z\partial_-z\right) + \partial_+X_k\partial_-X_k\right], \tag{2.10}$$

where

$$Y_4 = \frac{1}{2z}\left(x^mx_m + z^2 - 1\right), \quad Y_5 = \frac{1}{2z}\left(x^mx_m + z^2 + 1\right).$$

Here, x_m, z are coordinates on AdS_5, while X_k are the coordinates on S^5. This leads to [16, 19] ($i\tau = \tau_e$)

$$\mathcal{C}_3^{\tilde{\gamma}} = c_\Delta^d \int_{-\infty}^{\infty} \frac{d\tau_e}{\cosh^4(\kappa\tau_e)} \int_{-L}^{L} d\sigma \left(\kappa^2 + \mathcal{L}_f\right). \tag{2.11}$$

Performing the integrations in (2.11), one finds

$$\begin{aligned} \mathcal{C}_3^{\tilde{\gamma}} = & \frac{16}{3} c_\Delta^d \frac{1}{\sqrt{(1-u^2)W(\chi_p-\chi_n)}} \times \\ & \left[\left((1-u^2)(1-\tilde{\gamma}K) - \tilde{\gamma}uvW\right)\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right. \\ & \left. + \left(\left(W(1-\tilde{\gamma}uv\chi_n) - (1-\tilde{\gamma}K)\left(1-(1-u^2)\chi_n\right)\right)\mathbf{K}(1-\epsilon)\right)\right], \end{aligned} \tag{2.12}$$

where $\mathbf{K}(1-\epsilon)$ and $\mathbf{E}(1-\epsilon)$ are the complete elliptic integrals of first and second kind, and the following notation has been introduced

$$\epsilon = \frac{\chi_m - \chi_n}{\chi_p - \chi_n}. \tag{2.13}$$

This is our *exact* result for the normalized coefficient $\mathcal{C}_3^{\tilde{\gamma}}$ in the three-point correlation function, corresponding to the case when the heavy vertex operators are *finite-size* dyonic giant magnons living on the γ-deformed three-sphere.

For further purposes, let us also write down the exact expressions for the conserved charges and the angular differences

$$\mathcal{E} \equiv \frac{2\pi E}{\sqrt{\lambda}} = 2\frac{(1-v^2)\sqrt{W}}{\sqrt{1-u^2}} \frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}}, \tag{2.14}$$

$$\mathcal{J}_1 \equiv \frac{2\pi J_1}{\sqrt{\lambda}} = \frac{2}{\sqrt{1-u^2}} \left[\frac{1-\chi_n - v\,(vW-uK)}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) - \sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right], \tag{2.15}$$

$$\mathcal{J}_2 \equiv \frac{2\pi J_2}{\sqrt{\lambda}} = \frac{2}{\sqrt{1-u^2}} \left[\frac{u\chi_n - vK}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) + u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right], \tag{2.16}$$

$$\begin{aligned} p_1 \equiv \Delta\phi_1 = \phi_1(L) - \phi_1(-L) = & \frac{2}{\sqrt{1-u^2}} \\ \times\Bigg\{ & \frac{vW-uK}{(1-\chi_p)\sqrt{\chi_p-\chi_n}}\Pi\left(-\frac{\chi_p-\chi_m}{1-\chi_p}|1-\epsilon\right) - \left[v\,(1-\tilde{\gamma}K) + \tilde{\gamma}u\chi_n\right]\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}} \\ & -\tilde{\gamma}u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\Bigg\}, \end{aligned} \tag{2.17}$$

$$p_2 \equiv \Delta\phi_2 = \phi_2(L) - \phi_2(-L) = \frac{2}{\sqrt{1-u^2}} \tag{2.18}$$
$$\times\left\{\frac{K}{\chi_p\sqrt{\chi_p-\chi_n}}\Pi\left(1-\frac{\chi_m}{\chi_p}|1-\epsilon\right) - \left[uv+\tilde{\gamma}v\left(vW-uK\right)-\tilde{\gamma}\left(1-\chi_n\right)\right]\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}}\right.$$
$$\left.-\tilde{\gamma}\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right\}.$$

Here, E, $J_{1,2}$ are the string energy and angular momenta, while $\phi_{1,2}$ are the isometry angles on the γ-deformed three-sphere.

3 Small ϵ expansions

For the case of the dilaton operator, the three-point function of the SYM can be easily related to the conformal dimension of the heavy operators. This corresponds to shift 't Hooft coupling constant which is the overall coefficient of the Lagrangian [5]. This gives an important relation between the structure constant and the conformal dimension as follows:

$$\mathcal{C}_3^{\tilde{\gamma}} = \frac{32\pi}{3}c_\Delta^d\sqrt{\lambda}\partial_\lambda\Delta. \tag{3.1}$$

We want to show here that this relation holds for the case of finite-size giant magnons ($J_2 = 0$), assuming that $\Delta = E - J_1$, and considering the limit $\epsilon \to 0$. To this end, we introduce the expansions

$$\begin{aligned}
\chi_p &= \chi_{p0} + \left(\chi_{p1} + \chi_{p2}\log(\epsilon)\right)\epsilon, \\
\chi_m &= \chi_{m0} + \left(\chi_{m1} + \chi_{m2}\log(\epsilon)\right)\epsilon, \\
\chi_n &= \chi_{n0} + \left(\chi_{n1} + \chi_{n2}\log(\epsilon)\right)\epsilon, \\
v &= v_0 + \left(v_1 + v_2\log(\epsilon)\right)\epsilon, \\
u &= u_0 + \left(u_1 + u_2\log(\epsilon)\right)\epsilon, \\
W &= W_0 + \left(W_1 + W_2\log(\epsilon)\right)\epsilon, \\
K &= K_0 + \left(K_1 + K_2\log(\epsilon)\right)\epsilon.
\end{aligned} \tag{3.2}$$

A few comments are in order. To be able to reproduce the dispersion relation for the infinite-size giant magnons, we set

$$\chi_{m0} = \chi_{n0} = K_0 = 0, \quad W_0 = 1. \tag{3.3}$$

In addition, one can check that if we keep the coefficients χ_{m2}, χ_{n2}, W_2 and K_2 nonzero, the known leading correction to the giant magnon energy-charge relation [26] will be modified by a term proportional to $\mathcal{J}_1^2$. That is why we choose

$$\chi_{m2} = \chi_{n2} = W_2 = K_2 = 0. \tag{3.4}$$

Finally, since we are considering for simplicity giant magnons with one angular momentum, we also set

$$u_0 = 0, \tag{3.5}$$

because the leading term in the ϵ-expansion of $\mathcal{J}_2$ is proportional to u_0.

By replacing (3.2) in (2.9) and (2.13), and taking into account (3.3), (3.4), (3.5), we obtain

$$\begin{aligned}
\chi_{p0} &= 1 - v_0^2, \\
\chi_{p1} &= \frac{v_0}{1-v_0^2}\left[v_0\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)} - 2(1-v_0^2)v_1\right], \\
\chi_{p2} &= -2v_0 v_2, \\
\chi_{m1} &= \frac{(1-v_0^2)^2 + \sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}}{2(1-v_0^2)}, \\
\chi_{n1} &= -\frac{(1-v_0^2)^2 - \sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}}{2(1-v_0^2)}, \\
W_1 &= -\frac{\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}}{1-v_0^2}.
\end{aligned} \tag{3.6}$$

The other parameters in (3.2) and (3.6) can be found in the following way. First, we impose the conditions $J_2 = 0$ and p_1 to be independent of ϵ. This leads to four equations with solution

$$\begin{aligned}
v_1 &= \frac{v_0\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}\,(1-\log 16)}{4(1-v_0^2)}, \\
v_2 &= \frac{v_0\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}}{4(1-v_0^2)}, \\
u_1 &= \frac{K_1 v_0 \log 4}{1-v_0^2}, \\
u_2 &= -\frac{K_1 v_0}{2(1-v_0^2)},
\end{aligned} \tag{3.7}$$

where

$$v_0 = \cos\frac{p_1}{2}. \tag{3.8}$$

Next, to the leading order, the expansions for $\mathcal{J}_1$ and $p_2 = 2\pi n_2$ ($n_2 \in \mathbb{Z}$) give

$$\epsilon = 16 \exp\left(-2 - \frac{\mathcal{J}_1}{\sin\frac{p_1}{2}}\right), \qquad K_1 = \frac{1}{2}\sin^3\frac{p_1}{2}\sin\Phi, \qquad \Phi = 2\pi\left(n_2 - \frac{\tilde{\gamma}}{\sqrt{\lambda}}J_1\right). \tag{3.9}$$

Now, we consider the limit $\epsilon \to 0$ in the expression (2.12) for the structure constant in the 3-point correlation function, by using (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), and obtain

$$\begin{aligned}
\mathcal{C}_3^{\tilde{\gamma}} \approx\ & \frac{4}{3}c_\Delta^d \frac{1}{(1-v_0^2)^{3/2}}\Bigg[4 + 4v_0^4\Big(1 - \tilde{\gamma}K_1(1-\log 4)\ \epsilon\Big) \\
& -v_0^2\left(8 + \left(\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}\ (1-\log 16) - 8\tilde{\gamma}K_1(1-\log 4)\right)\epsilon\right) \\
& -\left(4\tilde{\gamma}K_1(1-\log 4) - \sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}\ (1-\log 256)\right)\epsilon \\
& -\left(v_0^2\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)} + 2\tilde{\gamma}K_1(1-v_0^2)^2\right)\epsilon\log\epsilon \\
& +\sqrt{(1-v_0^2)^4 - 4K_1^2(1-v_0^2)}\ \epsilon\log(16\ \epsilon)\Bigg].
\end{aligned} \tag{3.10}$$

According to (3.8), (3.9), the above expression for $\mathcal{C}_3^{\tilde{\gamma}}$ can be rewritten in terms of p_1, $\mathcal{J}_1$, as

$$\mathcal{C}_3^{\tilde{\gamma}} \approx \frac{16}{3}c_\Delta^d \sin\frac{p_1}{2}\left[1 - 4\sin^2\frac{p_1}{2}\left(\cos\Phi + \mathcal{J}_1\csc\frac{p_1}{2}\cos\Phi - \tilde{\gamma}\mathcal{J}_1\sin\Phi\right)e^{-2-\frac{\mathcal{J}_1}{\sin\frac{p_1}{2}}}\right] \tag{3.11}$$

In order to check if the equality (3.1) holds for the present case, let us now consider the dispersion relation of giant magnons on TsT-transformed $AdS_5 \times S^5$, including the leading finite-size correction, which is known to be [27, 28]

$$E - J_1 = \frac{\sqrt{\lambda}}{\pi}\sin(p/2)\left[1 - 4\sin^2(p/2)\cos\Phi\exp\left(-2 - \frac{2\pi J_1}{\sqrt{\lambda}\sin(p/2)}\right)\right]. \tag{3.12}$$

Taking the λ derivative of (3.12), one finds

$$\lambda\partial_\lambda\Delta = \frac{\sqrt{\lambda}}{2\pi}\sin\frac{p}{2}\left[1 - 4\sin^2\frac{p}{2}\left(\cos\Phi + \mathcal{J}_1\csc\frac{p}{2}\cos\Phi - \tilde{\gamma}\mathcal{J}_1\sin\Phi\right)e^{-2-\frac{\mathcal{J}_1}{\sin\frac{p}{2}}}\right]. \tag{3.13}$$

Identifying $p \equiv p_1$, and comparing (3.11) with (3.13), we see that the equality (3.1) is also valid for the γ-deformed case.

4 Concluding Remarks

In this note, we have derived the structure constant in the three-point correlation function of two finite-size (dyonic) giant magnon string states and a light dilaton state in the semi-classical approximation, for the case of γ-deformed (TsT-transformed) $AdS_5 \times S^5$, dual to $\mathcal{N} = 1$ SYM, arising as an exactly marginal deformation of $\mathcal{N} = 4$ SYM. We have confirmed our result by showing that the important relation between the structure constant and the derivative of the conformal dimension with respect to the t'Hooft coupling λ still holds for the γ-deformed case. It will be interesting to consider correlation functions of other light operators or even all the heavy string states in the future.

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